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EARLY TRANSIENT RESPONSE DURING CRACK PROPAGATION IN A WEAKLY-COUPLED THERMOELASTIC SOLID

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Abstract—A semi-infinite crack grows due to stress wave diffraction in a weakly-coupled thermoelastic solid. A rudimentary inelastic zone at the crack edge acts both as a heat flux site and as a crack-blunting mechanism. The transient analysis is first-step in the sense that constant crack/zone extension speeds are considered, and the material properties do not themselves vary with temperature.

The *a priori* unknown heat flux in the zone, the temperature response at the zone edge, the crack opening displacement and the rate of energy production in the zone are studied in the period right after fracture/zone initiation.

These expressions show that the applied (incident wave) stress should exceed the value necessary for zone formation predicted by a non-thermal analysis. The zone edge temperature increases rapidly at first, but then begins to level off in the same range of values noted for steady-state analyses at the time limit of the model's validity. The temperature rise varies inversely with zone growth rate, while the crack opening and energy production rate vary directly.

1. INTRODUCTION

Analytical, numerical and experimental results (Weichert and Schoenert, 1974; Parvin, 1979; Zehnder and Rosakis, 1991) indicate that significant temperature rises can occur during fracture. Because 90% of plastic deformation energy goes into heat (Taylor and Quinney, 1934) these temperature rises are associated with inelastic zone formation at the crack edges.

Studies, such as those cited above, often treat steady-state crack growth. In contrast, we are interested here in rapid, nearly brittle fracture under dynamic loading during the period right after initiation. A rudimentary elastic zone acts as a heat flux site on the boundary of a weakly-coupled thermoelastic solid (weak coupling will be defined later).

The solid is unbounded, the crack is semi-infinite, and the dynamic loading is provided by stress wave diffraction. As in studies such as those by Rice and Levy (1969) and Williams (1972), the inelastic zone will have a Dugdale (1960) geometry, and will cause crack blunting. However, the zone heat flux is not *a priori* fixed, and the fracture and heat energies are not necessarily the same. A transient two-dimensional analysis is performed, and nearly exact results obtained for short times after fracture/zone growth initiation. As a first step, a simple heat flux model and constant, subcritical crack and zone extension rates are treated.

Symmetry will be invoked to reduce the analysis to a mixed boundary/initial value problem in a half-plane. Therefore, in the next section, the general problem of a fully-coupled thermoelastic half-plane is examined, and integral transforms for the weakly-coupled case extracted. Boley and Tolins (1962) have, of course, considered the half-plane problem. Here, however, the focus will be on transform expressions of particular use in mixed problems.

2. TRANSIENT STUDY OF THERMOELASTIC HALF-PLANE

Consider the half-plane $y > 0$, where (x, y) are Cartesian coordinates. For $s < 0$, where s = (time × dilatational wave speed, v_1), the half-plane is at rest in a uniform temperature field T_0 (°K). For $s > 0$, we define θ as the change in temperature from T_0 and $u_i(x, y, s)$ as the change in the rest value of the i-displacement, where $i = (x, y)$. That is,

$$(u_i, \theta) = 0 \tag{1}$$

for $y > 0$, $s \leqslant 0$ while, by linear superposition, the governing equations

$$(m^2-1)\Delta_{,i}+\nabla^2 u_i+\beta\theta_{,i}-m^2\ddot{u}_i = 0, \quad i = (x, y) \tag{2a}$$

$$\frac{K}{\mu}\nabla^2\theta - c_v\frac{\alpha}{v_2}\dot{\theta}+\beta T_0 v_1\dot{\Delta} = 0 \tag{2b}$$

and

$$\frac{1}{\mu}\sigma_{ij} = [\beta\theta+(m^2-2)\Delta]\delta_{ij}+2(u_{i,j}+u_{j,i}), \quad (i,j) = (x,y) \tag{3}$$

hold for $y > 0$, $s > 0$. Here σ_i denotes identical indices, and

$$m = v_1/v_2 > 1, \quad \beta = \beta_0(4-3m^2) < 0 \tag{4}$$

where β has the dimensions of inverse temperature (°K^{-1}). The symbol ∇^2 is the Laplacian operator, Δ is the dilatation, ($\dot{}$) denotes s-differentiation and $(\ )_{,i}$ denotes i-differentiation. The material constants $(\beta_0, K, \mu, v_2, c_v)$ are, respectively, the coefficient of linear expansion, the thermal conductivity, the shear modulus, the rotational wave speed, and the specific heat at constant deformation. Equations (2) are valid (Boley and Weiner, 1960) under the assumption that $|\theta| \ll |T_0|$ almost everywhere.

To solve (1) and (2), the unilateral and bilateral Laplace transforms

$$F = \int_0^\infty f(s)\,\mathrm{e}^{-ps}\,\mathrm{d}s, \quad F^* = \int_{-\infty}^{\infty} F(x)\,\mathrm{e}^{-pqx}\,\mathrm{d}x \tag{5a,b}$$

(Sneddon, 1972) are applied, where p here can be treated as real and positive but q is, in general, complex. Under the reasonable assumptions that (u_i, θ) are continuous across elastic wavefronts, and vanish as $\sqrt{(x^2+y^2)} \to \infty$ for finite s, (2) and (1) reduce to the symmetric coupled ODE set

$$\begin{bmatrix} \partial^2-m^2p^2a_0^2 & (m^2-1)pq\partial & \beta pq \\ (m^2-1)pq\partial & m^2\partial^2-p^2b_0^2 & \beta\partial \\ \beta pq & \beta\partial & \frac{\gamma}{p}(\partial^2+p^2q^2)-\gamma_0 \end{bmatrix} \begin{bmatrix} U_x^* \\ U_y^* \\ \Theta^* \end{bmatrix} = 0 \tag{6}$$

where ∂ denotes y-differentiation and

$$\gamma = \frac{K}{\mu v_1 T_0}, \quad \gamma_0 = \frac{c_v}{v_2^2 T_0}, \quad a_0 = \sqrt{(1-q^2)}, \quad b_0 = \sqrt{(m^2-q^2)}. \tag{7}$$

Here $\mathrm{Re}(a_0, b_0) \geqslant 0$ in the q-planes cut along $\mathrm{Im}(q) = 0$, $|\mathrm{Re}(q)| > (1, m)$. Solutions bounded for $y > 0$ and valid for $hp > 1$ are

$$U_x^* = B\,e^{-pb_0 y} + A_\varepsilon X_+ A\,e^{-pa_+ y} + X_- C\,e^{-pa_- y} \tag{8a}$$

$$U_y^* = \frac{q}{b_0} B\,e^{-pb_0 y} + A_\varepsilon Y_+ A\,e^{-pa_+ y} + Y_- C\,e^{-pa_- y} \tag{8b}$$

$$\Theta^* = A_\varepsilon A\,e^{-pa_+ y} + C\,e^{-pa_- y} \tag{8c}$$

where (A, B, C) are arbitrary functions of (p, q) and, for the case $hp < 1$, the subscripts $\pm$ are interchanged. In either case,

$$A_\varepsilon = -\frac{m^2 p\varepsilon}{\beta q(hp-1)}, \quad X_\pm = \frac{\beta q}{\varepsilon p}\frac{\Omega_\pm}{m^2 - M_\pm^2}, \quad Y_\pm = \frac{\beta}{\varepsilon p a_\pm}\frac{\varepsilon q^2 + (a_0^2 - a_\pm^2/m^2)K_\pm}{m^2 - M_\pm^2} \tag{9a}$$

$$\Omega_\pm = \varepsilon + \left(\frac{1}{m^2} - 1\right)K_\pm, \quad K_\pm = \frac{1}{\tau}M_\pm^2 - 1, \quad a_\pm = \sqrt{(M_\pm^2 - q^2)} \tag{9b}$$

$$\tau = \frac{1}{hp}, \quad M_\pm^2 = \tfrac{1}{2}[1 + (1+\varepsilon)\tau] \pm \sqrt{(\tfrac{1}{4}[1-(1+\varepsilon)\tau]^2 + \varepsilon\tau)} \tag{9c}$$

where $\mathrm{Re}\,(a_\pm) \geq 0$ in the q-planes cut along $\mathrm{Im}\,(q) = 0$, $|\mathrm{Re}\,(q)| > M_\pm$ and

$$h = \frac{\gamma}{\gamma_0}, \quad \varepsilon = \frac{1}{\gamma_0}\left(\frac{\beta}{m}\right)^2. \tag{10a,b}$$

Here h is the thermoelastic characteristic length, and the dimensionless parameter ε is the coupling constant (Chadwick, 1960; Achenbach, 1973). For many thermoelastic materials $\varepsilon \leq 0(10^{-2})$, and it is noted that here it often occurs—such as in the key parameter $M_\pm$—in sums or differences with dimensionless numbers of 0(1). We here, therefore, define a weakly-coupled thermoelastic solid as one for which ε in such circumstances is viewed as a perturbation and can be neglected. By starting with $M_\pm$, it can then be shown that so long as

$$S\varepsilon < 0(1), \quad S = \frac{hp+1}{(hp-1)^2} \tag{11}$$

an excellent approximation to (8) for both ($hp > 1$, $hp < 1$) is

$$U_x^* = B\,e^{-pb_0 y} + A\,e^{-pa_0 y} + \frac{\beta q h}{m^2(hp-1)} C\,e^{-p\alpha_0 y} \tag{12a}$$

$$U_y^* = \frac{q}{b_0} B\,e^{-pb_0 y} - \frac{a}{q} A\,e^{-pa_0 y} - \frac{\beta\alpha h}{m^2(hp-1)} C\,e^{-p\alpha_0 y} \tag{12b}$$

$$\Theta^* = C\,e^{-p\alpha_0 y} \tag{12c}$$

where (A, B, C) are as before, while

$$\alpha_0 = \sqrt{(\tau - q^2)} \tag{13}$$

and $\mathrm{Re}\,(\alpha_0) \geq 0$ in the plane cut along $\mathrm{Im}\,(q) = 0$, $|\mathrm{Re}\,(q)| > \sqrt{\tau}$.

The results (12) are identical in form to those that would be obtained if (6) were solved after dropping the β-terms in the bottom row of the matrix. That is, the β-term in the temperature diffusion equation (2b) can be ignored. Thus, the temperature field in a weakly-coupled thermoelastic solid affects the kinematical field equations, but not vice-versa. Such behavior is analogous to that for a solid in elastostatics (Boley and Weiner, 1960). The a_0-terms in (12) suggest that the weakly-coupled solid propagates standard (Achenbach, 1973) dilatational waves.

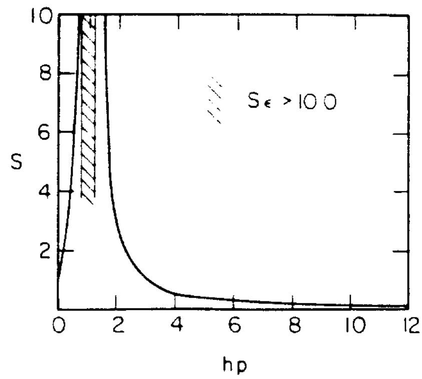

Fig. 1. Transform variable expansion parameter.

A plot of S vs $hp \geqslant 0$ in Fig. 1 shows that for $\varepsilon \sim 0(10^{-2})$, (11) is satisfied except in a range roughly defined by $0.75 < hp < 1.25$. Thus, (12) is an adequate double transform representation for very small and very large values of hp. From the Abelian theorems of Laplace transforms (Noble, 1958; Sneddon, 1972) it follows that the functions themselves should be very accurate for short and long times. The former situation is, as noted at the outset, of interest to us here.

With the mathematics of the weakly-coupled solid established, the crack problem can be attacked. However, it will be more convenient to work with the coordinates (ξ, y, s), where

$$\xi = x - cs, \quad u_\xi = u_x \tag{14a,b}$$

and c is a constant speed non-dimensionalized with respect to c_1. For the weakly-coupled solid, (2) is in view of (7) and (10a) replaced by

$$(m^2-1)\Delta_{,i} + \nabla^2 u_i + \beta\theta_{,i} - m^2(\ddot{u}_i - 2c\dot{u}_{i,\xi} + c^2 u_{i,\xi\xi}) = 0, \quad i = (\xi, y) \tag{15a}$$

$$h\nabla^2\theta + c\theta_{,\xi} - \dot{\theta} = 0 \tag{15b}$$

where (Δ, ∇^2) are now defined in terms of (ξ, y). As seen in (15), the transformation (14a) preserves three independent variables (ξ, y, s). Similar transformations (Sneddon and Lowengrub, 1969) lead to steady-state equations in the two variables (ξ, y) by suppressing s-derivatives.

Double transforms are now with respect to (ξ, s), and (5b) is replaced by

$$F^* = \int_{-\infty}^{\infty} F(\xi)\, e^{-pq\xi}\, d\xi. \tag{16}$$

In view of (1), the application of (5a) and (16) to (15) gives

$$U_x^* = B\, e^{-pby} + A\, e^{-pay} + \frac{\beta q}{m^2 p(a^2-\alpha^2)} C\, e^{-p\alpha y} \tag{17a}$$

$$U_y^* = \frac{q}{b} B\, e^{-pby} - \frac{a}{q} A\, e^{-pay} - \frac{\beta\alpha}{m^2 p(a^2-\alpha^2)} C\, e^{-p\alpha y} \tag{17b}$$

$$\Theta^* = C\, e^{-p\alpha y} \tag{17c}$$

where now

$$a = \sqrt{[(1-cq)^2-q^2]}, \quad b = \sqrt{[m^2(1-cq)^2-q^2]}, \quad \alpha = \sqrt{[\tau(1-cq)-q^2]} \tag{18}$$

and $\operatorname{Re}(a, b, \alpha) \geqslant 0$ in planes with the branch cuts

$$a: \quad \operatorname{Im}(q) = 0, \quad i_- > \operatorname{Re}(q), \operatorname{Re}(q) > i_+; \quad i_\pm = \pm 1/(1 \pm c) \tag{19a}$$

$$b: \quad \operatorname{Im}(q) = 0, \quad m_- > \operatorname{Re}(q), \operatorname{Re}(q) > m_+; \quad m_\pm = \pm m/(1 \pm mc) \tag{19b}$$

$$\alpha: \quad \operatorname{Im}(q) = 0, \quad \tau_- > \operatorname{Re}(q), \operatorname{Re}(q) > \tau_+; \quad \tau_\pm = \sqrt{\tau\left[-\frac{c}{2}\sqrt{\tau} \pm \sqrt{\left(1+\frac{c^2}{4}\tau\right)}\right]}. \tag{19c}$$

Clearly, (12) is recovered when $c = 0$. The sequel will call for the double transforms of the tractions $(\sigma_{\xi y}, \sigma_y)$. These expressions can be obtained from (3) with x replaced by ξ, (16) and (17) as

$$\frac{1}{\mu}\Sigma^*_{\xi y} = -p\frac{N}{b}B\,\mathrm{e}^{-pby} - 2paA\,\mathrm{e}^{-pay} - \frac{2Bq\alpha}{m^2(a^2-\alpha^2)}C\,\mathrm{e}^{-p\alpha y} \tag{20a}$$

$$\frac{1}{\mu}\Sigma^*_y = -2pqB\,\mathrm{e}^{-pby} + p\frac{N}{q}A\,\mathrm{e}^{-pay} + \frac{\beta N}{m^2(a^2-\alpha^2)}C\,\mathrm{e}^{-p\alpha y} \tag{20b}$$

where

$$N = m^2(1-cq)^2 - 2q^2. \tag{21}$$

The crack problem is stated in the next section. It will be seen that, despite the weak field equation coupling, determination of θ does depend on u_i because of inelastic zone-imposed boundary conditions.

3. THE CRACK PROBLEM

Consider the semi-infinite crack $y = 0$, $x < 0$ in an unbounded, weakly-coupled solid. For $s < 0$, equilibrium exists for a uniform temperature field T_0. That is, the undisturbed crack is essentially closed, so that a uniform temperature field is possible. A plane dilatational wave defined by

$$u_x = 0, \quad \mu m^2 u_y = -\int_0^{s-y} \sigma(t)\,\mathrm{d}t, \quad s \geqslant y \tag{22}$$

is, however, traveling through the material at right angles to the crack plane. Here σ is the traction σ_y generated a distance t behind the wavefront. When (2b) with the β-term dropped or (15b) hold, it is easily shown that (22) is associated with the constant θ, which we will take to be zero. At $s = 0$, the wave reaches the crack, and is diffracted at its edge.

We assume that almost instantaneously the crack extends along the positive x-axis, and that a vanishingly thin elastic zone forms simultaneously ahead of it. The geometry of the diffraction/fracture process and the accompanying wave pattern are shown schemati-

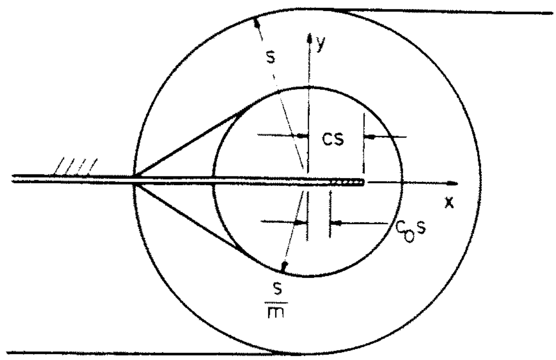

Fig. 2. Crack propagation process and associated wave pattern.

cally in Fig. 2. There (c, c_0) are the constant, subcritical speeds of the inelastic zone and crack edge, non-dimensionalized with respect to c_1, where

$$c_0 < c < c_R \tag{23}$$

and c_R is the non-dimensionalized Rayleigh wave speed.

Although the inelastic zone is close to vanishing, it behaves as a perfectly plastic material (Dugdale, 1960). Therefore, since the plane wave (22) induces no shear σ_{xy} in the plane of the zone, we should have

$$y = 0^+, \quad c_0 s < x < cs: \quad \sigma_{xy} = 0, \sigma_y = Y \tag{24}$$

where Y is the yield stress. If indeed a significant temperature rise occurs, then Y should vary inversely with θ (Boyer and Gall, 1985). For mathematical simplicity in this first-step analysis, we keep Y constant. Because the inelastic zone acts as a heat flux site, we expect that

$$y = 0\pm, \quad c_0 s < x < cs: \quad \partial\theta = \pm\psi_0(x - c_0 s) \tag{25}$$

where ψ_0 is the flux function, and must be determined. As a first step, dependence only on the distance from the crack edge has been assumed.

The solution for this problem can be viewed as the sum of the plane wave field (22) (with $\theta = 0$) and the T_0-induced static field with the (u_i, θ) resulting from the removal of the (22)-induced tractions from the surfaces of an identical crack growing in an initially undisturbed solid. Since the former two fields are, in effect, known, attention focuses on the traction removal problem: this problem exhibits crack plane symmetry, so that only the half-plane $y > 0$ need be considered by imposing, in view of (22), (24) and (25), the conditions

$$\sigma_{xy} = 0, \quad \sigma_y = YH(x - c_0 s) - \sigma(s)(x < cs), \quad u_y = 0(x \geqslant cs) \tag{26a}$$

$$\partial\theta = \psi_0(x - c_0 s)(c_0 s < x < cs), \quad \partial\theta = 0(x \geqslant cs, x \leqslant c_0 s) \tag{26b}$$

along $y = 0$, where $H(\ )$ is the Heaviside function. For $y > 0$, $s \leqslant 0$,

$$(u_i, \theta) \equiv 0 \tag{27}$$

while for $(y, s) > 0$, (2) and (3) hold, with the β-term in (2b) suppressed. In addition, wavefront continuity and boundedness as $\sqrt{(x^2 + y^2)} \to \infty$ for finite s for (u_i, θ) is expected.

To study this problem, we employ the Wiener–Hopf (Noble, 1958) technique. Therefore, (14) is introduced, and (15) becomes the field equation for $(y, s) > 0$. Conditions (26a,b) take the form

$$\partial\theta = \psi(s+k\xi)H(s+k\xi), \quad u_y = u_-(\xi,s)H(-\xi) \tag{28a,b}$$

$$\sigma_y = YH(s+k\xi) - \sigma(s)H(-\xi) + \sigma_+(\xi,s)H(s-v\xi)H(\xi), \quad \sigma_{\xi y} = 0 \tag{28c,d}$$

where v is a vanishingly small, dimensionless positive constant, and

$$k = \frac{1}{\delta c}, \quad \delta c = c - c_0, \quad \psi_0(s) = \psi(ks), \quad u_-(0-,s) = 0. \tag{29a–d}$$

Here (σ_+, u_-) are, respectively, the unknown traction σ_y in the crack plane ahead ($\xi > 0$) of the inelastic zone, and the unknown u_y induced for $y = 0$, $\xi < 0$ by the crack opening and inelastic zone displacement discontinuity. Their introduction allows conditions on all four quantities $(\sigma_{\xi y}, \sigma_y, \partial\theta, u_y)$ to be stated everywhere on $y = 0$. The restriction (29d) recognizes that the inelastic zone displacement discontinuity vanishes at the zone edge. The argument of the Heaviside function with σ_+ recognizes that part of this traction radiates from the inelastic zone edge as elastic waves (Achenbach, 1973) while part is, due to the parabolic nature of (15b), established everywhere instantaneously, but decays exponentially for finite s as $|\xi| \to \infty$ (Carrier and Pearson, 1988). That is, σ_+ can be viewed as essentially zero at some time-dependent finite distance away from the crack edge. It will be seen that v actually does not appear in the solution.

Application of (5a) and (16) to (28) yields the double transform conditions

$$\partial\Theta^* = \frac{\Psi}{p(k-q)}, \quad U_y^* = U^*_-, \quad \Sigma_y^* = \frac{Y}{p^2(k-q)} + \frac{\Sigma}{pq} + \Sigma_+^*, \quad \Sigma_{\xi y}^* = 0 \tag{30a–d}$$

along $y = 0$. The right-hand side of (30a) and the first term in (30c) are analytic for $\text{Re}(q) < k$, while the right-hand side of (30b) and the second term in (30c) are analytic for $\text{Re}(q) < 0$. The last term in (30c) is analytic for $\text{Re}(q) > -v$. Double transforms that satisfy wavefront continuity, boundedness, (28) and the field equations (15) for $y > 0$ have already been obtained. Therefore our problem can be solved merely by identifying the parameter c in those results, (17) and (20), with the dimensional zone speed c here, and then enforcing (30). If for the moment Ψ is treated as given, then (17) and (20) contain three arbitrary parameters (A, B, C), while the four conditions imposed by (30) involve the two unknowns (U_-^*, Σ_+^*). Equations (30) can be used, therefore, to eliminate (A, B, C) and reduce the analysis to the single Wiener–Hopf equation

$$\frac{\beta N}{p^2 \alpha R}\frac{(1-cq)^2}{\alpha + a}\frac{\Psi}{k-q} + m^2 a\frac{(1-cq)^2}{\mu R}\left[\frac{Y}{p^2(k-q)} + \frac{\Sigma}{pq} + \Sigma_+^*\right] = -pU_-^* \tag{31}$$

for (U_-^*, Σ_+^*). Here

$$R = 4q^2 ab + N^2 \tag{32}$$

is a form of the Rayleigh function, with zeroes at $q = n_\pm$, where

$$n_\pm = \pm n/(1 \pm nc), \quad n = 1/c_R. \tag{33}$$

4. SOLUTION OF WIENER–HOPF EQUATION

The regions of analyticity noted earlier show that various terms in (31) are analytic in certain areas of the q-plane. This suggests that (31) be rewritten in such a way that the two sides of the equation are analytic in overlapping half-planes. As a first step toward achieving this end, we note (Freund, 1971) that R can be written as

$$\frac{R}{(1-cq)^2} = \lambda G_+(q)(q-n_-)G_-(q)(q-n_+) \tag{34}$$

where

$$\lambda = \frac{1}{c^2} R_0(c), \quad R_0(c) = 4\sqrt{(1-c^2)}\sqrt{(1-m^2c^2)} - (2-m^2c^2)^2 \tag{35a}$$

$$\pi \ln G_\pm(q) = -\int_1^m \frac{\Omega}{\dfrac{t}{1\mp ct} \pm q} \frac{dt}{(1\mp ct)^2} \tag{35b}$$

$$\Omega = \tan^{-1} \frac{4t^2|a_0(t)|b_0(t)}{N_0^2(t)}, \quad N_0(t) = m^2 - 2t^2. \tag{35c}$$

The first pair of q-dependent terms on the right-hand side of (34) is analytic for $\mathrm{Re}(q) > n_-$ while the last pair is analytic for $\mathrm{Re}(q) < n_+$. Similarly,

$$a = a_+ a_-, \quad a_\pm = \sqrt{(1-cq\pm q)} \tag{36}$$

where the factors $a_\pm$ are analytic in, respectively, the half-planes $\mathrm{Re}(q) > i_-$ and $\mathrm{Re}(q) < i_+$. In view of (34) and (36), (31) becomes

$$\frac{-m^2 a_+}{\mu G_+(q-n_-)} \Sigma_+^* = \lambda p \frac{G_-}{a_-}(q-n_+)U_-^* + \frac{m^2 a_+}{\mu p G_+(q-n_-)}\left[\frac{\Sigma}{q} + \frac{Y}{p(k-q)}\right] - \frac{\beta\Psi}{p^2} P \tag{37}$$

where

$$P = \frac{N}{(k-q)(\alpha^2-a^2)} \frac{1}{G_+(q-n_-)}\left(\frac{a_+}{\alpha} - \frac{1}{a_-}\right). \tag{38}$$

The left-hand side of (37) and the first term on the right-hand side are analytic for, respectively, $\mathrm{Re}(q) > -v$, $\mathrm{Re}(q) < 0$. The remaining terms can also be expressed as the sums of functions with overlapping half-planes of analyticity, either by inspection or by adopting a formal decomposition procedure (Noble, 1958). The result is

$$\frac{-m^2 a_+}{\mu G_+(q-n_-)} \Sigma_+^* - \frac{m^2\Sigma}{\mu p q}\left[\frac{a_+}{G_+(q-n_-)} + \frac{1}{n_- G_+(0)}\right] + \frac{\beta\Psi}{p^2} P_+$$

$$- \frac{m^2 Y}{\mu p^2(k-q)}\left[\frac{a_+}{G_+(q-n_-)} - \frac{a_+(k)}{G_+(k)(k-n_-)}\right] = \lambda G_- \frac{p}{a_-}(q-n_+)U_-^* - \frac{m^2\Sigma}{\mu p q n_- G_+(0)}$$

$$+ \frac{m^2 Y}{\mu p^2(k-q)} \frac{a_+(k)}{G_+(k)(k-n_-)} - \frac{\beta\Psi}{p^2}(P-P_+). \tag{39}$$

Here, the q-dependence of (a_+, G_+) is understood unless specified otherwise, and the form of P_+ depends on where the branch points $\tau_\pm$ are in relation to the other branch points and singularities of P. For the case $hp \gg 1$ which is of interest here, we have $i_- < \tau_- < 0$, $0 < \tau_+ < k$ and, therefore

$$\pi P_+ = \int_{\tau_+}^{1} \frac{P_\tau(w)\,dw}{w+q(1-cw)} + \int_1^m \frac{P_1(w)\,dw}{w+q(1-cw)} + \frac{\pi P_n(n)}{n+q(1-nc)}. \tag{40}$$

In (40)

$$P_\tau(w) = F_+\left(\frac{1}{c}\right) N_0 \frac{\delta c(1-nc)\sqrt{(1-w)}\sqrt{(1-cw)^3}}{(1-c_0 w)F_-(n-w)|\alpha|[\tau(1-cw)-1]} \tag{41a}$$

$$P_1(w) = \frac{4\lambda\delta c}{1-nc} N_0 |a_0| b_0 \frac{w^2(w+n)F_+}{(1-c_0 w)D[\tau(1-cw)-1]} \frac{1}{F_+\left(\frac{1}{c}\right)} \left[\frac{\sqrt{(w-1)}}{|\alpha|} + \frac{1}{\sqrt{(w+1)}}\right] \tag{41b}$$

$$P_n(n) = N_0(n)\delta c F_-\left(\frac{1}{c}\right) \frac{\sqrt{(1-nc)^5}}{(1-c_0 n)[\tau(1-nc)-1]} \frac{1}{F_-(n)} \left[\frac{\sqrt{(n-1)}}{|\alpha(-n)|} - \frac{1}{\sqrt{(n+1)}}\right] \tag{41c}$$

where

$$D = |R_0|^2, \quad \pi \ln F_\pm(w) = -\int_1^m \frac{\Omega\,dt}{t \pm w} \tag{42a,b}$$

and the w-dependence is understood unless specified otherwise. In writing (41), the effects of the speed parameters (c, c_0) were made more explicit by using (29a), appropriate integration variable changes and, from (35b) and (42b), the relation

$$G_\pm(q) = F_\pm\left(\frac{q}{1-cq}\right) \Big/ F_\mp\left(\frac{1}{c}\right). \tag{43}$$

The (Σ_+^*, Ψ)-terms on the left-hand side of (39) are analytic in, respectively, the regions $\mathrm{Re}(q) > (-v, \tau_-)$, while the (Σ, Y)-terms are analytic for $\mathrm{Re}(q) > i_-$. On the right-hand side, the (U_-^*, Σ)-terms are analytic for $\mathrm{Re}(q) < 0$ while the (Y, Ψ)-terms are analytic for, respectively, $\mathrm{Re}(q) < (k, \tau_+)$. That is, the left- and right-hand sides of (39) are analytic in the overlapping half-planes $\mathrm{Re}(q) > -v$ and $\mathrm{Re}(q) < 0$. They can then be viewed as the analytic continuations of the same entire function. The Abelian theorems and (30b) demonstrate that for $y = 0$, $\xi = 0-$

$$U_y \sim \lim_{|q|\to\infty} pqU_-^* \tag{44}$$

but (29d) implies that the left-hand side of (44) is zero. Therefore, U_-^* must behave as $0(q^{-1})$ when $|q| \to \infty$. A check of each term then shows that the right-hand side of (39) vanishes when $|q| \to \infty$. Thus, the entire function is bounded for all q, and Liouville's theorem states that such functions must be constant. Both sides of (39) are zero, therefore, and can be solved separately for (Σ_+^*, U_-^*).

5. CRACK PLANE STRESS AND AN EQUATION FOR ψ

With (Σ_+^*, U_-^*) available, the transformed crack problem is now in essence solved for $hp \gg 1$. A solution to the problem itself valid for small s can be obtained by inversion of these transforms. Of particular interest is the crack plane traction σ_+ very near the inelastic zone edge ($\xi \sim 0+$). For this case, the Abelian theorems imply that knowledge of Σ_+^* for large $|q|$ is sufficient. From (35)–(41) it can be shown that

$$\Sigma_+^* \sim \left[\frac{\mu\beta\Psi}{m^2 p} P_0 + \frac{\Sigma}{n_- G_+(0)} - \frac{Ya_+(k)}{pG_+(k)(k-n_-)}\right] \frac{1}{p\sqrt{(1-c)}\sqrt{q}} \tag{45}$$

for large q, where

$$\pi P_0 = \int_{-\infty}^{1} P_x(w)\frac{\mathrm{d}w}{1-cw} + \int_{1}^{m} P_1(w)\frac{\mathrm{d}w}{1-cw} + P_n(n)\frac{\pi}{1-nc}. \tag{46}$$

The inversion operations for the transforms (16) and (5a) are

$$F(\xi) = \frac{p}{2\pi i}\int_{\Gamma_q} F^* \mathrm{e}^{pq\xi}\,\mathrm{d}q, \quad f(s) = \frac{1}{2\pi i}\int_{\Gamma_p} F\,\mathrm{e}^{ps}\,\mathrm{d}p \tag{47a,b}$$

(Sneddon, 1972). Here Γ_q is a continuous path from $\mathrm{Im}(q) = -\infty$ to $\mathrm{Im}(q) = \infty$ that lies to the right of singularities and non-analytic behavior of F, while the contour Γ_p can here be taken along the left-hand side of the $\mathrm{Im}(q)$-axis. Substitution of (47a) and deformation of the Γ_q-contour by use of the Cauchy theorem to a path around the branch cut $\mathrm{Im}(q)$, $\mathrm{Re}(q) < 0$ of Σ_+^* gives an integral that can be performed with standard tables (Peirce and Foster, 1957). The result is

$$\Sigma_+ \sim \frac{1}{\sqrt{p}\sqrt{(1-c)}}\left[\frac{\mu\beta\Psi}{m^2 p} P_0 + F_-\left(\frac{1}{c}\right)\frac{(1-nc)\Sigma}{nF_+(0)} - F_-\left(\frac{1}{c}\right)\sqrt{(1-c_0)} \times \frac{(1-nc)Y\sqrt{\delta c}}{F_-\left(\frac{1}{c_0}\right)(1-c_0 n)p}\right]\frac{1}{\sqrt{(\pi\xi)}} \tag{48}$$

for $\xi \sim 0+$. We have again made (c, c_0) more explicit through the use of (29a) and (43). Because the field (22) and that induced by T_0 are well-behaved everywhere, (48) indicates that the inelastic zone edge stress is singular. Indeed, the $1/\sqrt{\xi}$-coefficient in (48) is the transform of the dynamic stress intensity factor. For crack blunting to occur (Dugdale, 1960), this factor must vanish, so that (48) gives the result

$$\Psi = F_-\left(\frac{1}{c}\right)\frac{1-nc}{\mu\beta P_0 n F_-(0)}(\omega Y - p\Sigma) \tag{49}$$

valid for $hp \gg 1$, where

$$\omega = F_-(0)\sqrt{(1-c_0)}\,\frac{n\sqrt{\delta c}}{(1-c_0 n)F_-\left(\frac{1}{c_0}\right)}. \tag{50}$$

Indeed, because we are interested in inelastic zone response right after fracture initiation, a Ψ valid for $hp \sim \infty$ suffices. To extract this expression is tedious, due to the factor P_0. Nevertheless, it can be shown that

$$\Psi \sim \frac{\pi}{\mu\beta \ln\sqrt{(hp)}}\frac{1}{\delta c}\left(1 - \frac{1}{hp}\right)(\omega Y - p\Sigma). \tag{51}$$

By using (47b) and the convolution theorem, the inverse of (51) is obtained as

$$\frac{\mu\beta\delta c}{2\pi}\psi(s) \sim -\frac{\omega Y}{h}L(s)+\int_0^s \dot{L}(s-t)\left[\frac{\sigma(t)}{h}-\dot{\sigma}(t)\right]dt \tag{52}$$

where ψ_0 in (25) follows by replacing s with $(x-c_0 s)/\delta c$, and

$$L(t) = \int_0^\infty \frac{du}{u}\frac{e^{-ut}}{(\ln hu)^2+\pi^2} \tag{53}$$

can be written in terms of tabulated functions (Gradshteyn and Ryzhik, 1980). Equations (25), (29c), (50) and (52) show that the heat flux generated in the inelastic zone right after fracture initiation varies inversely with zone growth rate δc, and appropriately vanishes at the crack edge.

With (52), the crack problem for small s is essentially solved to within determination of the dimensionless speeds (c, c_0). That requires a thermodynamically-based fracture criterion, but we can still study the temperature field θ for small s by treating (c, c_0) as given.

6. DETERMINATION OF θ

Returning to the process that led to (31), it can be shown that

$$\Theta^* = \frac{-\Psi}{p^2\alpha(k-q)}e^{-p\alpha y}. \tag{54}$$

Application of (47a) to (54) gives

$$\Theta = \frac{-\Psi}{2\pi i p}\int_{\Gamma_q} e^{p(q\xi-\alpha y)}\frac{dq}{\alpha(k-q)} \tag{55}$$

where, for large enough p, the singularity k lies on the integrand branch cut $\mathrm{Im}(q) = 0$, $\mathrm{Re}(q) > \tau_+$. Although the radical α is itself a function of p, it is advantageous to mimic the Cagniard–de Hoop (1960) method in developing (55). Thus, a contour in the q-plane is sought along which the exponential assumes the form e^{-pt}, where t is real and positive. Such a contour is defined as

$$rq_\pm(t) = -t\cos\phi - cr\tau\sin^2\phi \pm i\sin\phi\sqrt{(t-t_+)}\sqrt{(t-t_-)} \tag{56}$$

where

$$t_\pm = \sqrt{\tau}\left(\frac{\xi c}{2}\sqrt{\tau}\pm r\rho\right)(t_+ > t_-),\quad \rho = \sqrt{\left(1+\frac{c^2\tau}{4}\right)} \tag{57a}$$

$$r = \sqrt{(\xi^2+y^2)},\quad \xi = r\cos\phi,\quad y = r\sin\phi. \tag{57b}$$

For large enough p, $t_\pm$ are both real, with $t_+ > 0$. Then, (56) describes a Cagniard contour: two branches of a hyperbola which crosses the Re (q)-axis between the branch points $q = \tau_\pm$ of α. The Cauchy theorem can be used to change the path Γ_q onto this contour, and for the case $\xi > 0$ $(0 < \phi < \pi/2)$ we have

$$\Theta = \frac{-\Psi}{\pi}\mathrm{Re}\frac{1}{p}\int_{t_+}^\infty \frac{e^{-pt}}{\sqrt{(t-t_+)}\sqrt{(t-t_-)}}\frac{dt}{k-q_+(t)}. \tag{58}$$

The integration variable change $2t = (t_+-t_-)u+t_++t_-$ then yields

$$\Theta = \frac{-\Psi}{\pi} \operatorname{Re} \frac{e^{-(cr^2/2h)\cos\phi}}{p} \int_1^{\infty} \frac{e^{-r\sqrt{(p/h)}\rho u}}{\sqrt{(u^2-1)}} \frac{du}{k - q_+[t(u)]}. \tag{59}$$

The exponentials show clearly the expected decay of θ (and, thus, θ) as r increases. Of interest, therefore, is the behavior of θ at $r = 0$, the inelastic zone edge itself. Setting $r = 0$ in (56), (57) and (59) gives

$$\Theta_0 = -\frac{\Psi}{\pi} \operatorname{Re} \frac{\rho_0}{p} \int_1^{\infty} \frac{1}{\sqrt{(u^2-1)}} \frac{du}{k - q_0(u)} \tag{60}$$

where the subscript denotes evaluation at $r = 0$ and

$$q_0(u) = k + \frac{c\tau}{2} + \rho\sqrt{\tau}[u \cos\phi - i\sqrt{(u^2-1)} \sin\phi]. \tag{61}$$

The integrations in (60) can be performed with standard tables, so that for $hp \sim \infty$ we have

$$\Theta_0 \sim \frac{-\Psi}{\pi k p} \ln\left(\frac{2k}{\sqrt{\tau}}\right). \tag{62}$$

Substitution of (51) and application of (47b) gives, finally,

$$\theta_0 \sim 2 \frac{\ln(2k)}{\mu\beta} \int_0^s L(s-t)\left[\frac{1}{h}\sigma(t) - \dot{\sigma}(t) - \frac{\omega}{h} Y\right] dt. \tag{63}$$

For a step-stress incident wave

$$\sigma(s) = \sigma_0 (0 < \sigma_0 \ll Y) \tag{64}$$

(29a,b), (53) and (63) combine to give

$$\theta_0 \sim \frac{-2}{\mu\beta h} \ln\left(\frac{2}{\delta c}\right)(\sigma_0 - \omega Y) \int_0^s L(t)\, dt. \tag{65}$$

As indicated by (4) and (53), β is negative while the integral is positive. Therefore, a temperature increase at the inelastic zone edge right after fracture initiation will not occur unless

$$\frac{\sigma_0}{Y} > \omega. \tag{66}$$

Indeed, a strict equality, i.e. θ_0 vanishes, gives the same result as that required for fracture zone growth without thermal effects, *cf.* Nuismer and Achenbach (1972).

For the present situation, insight into the σ_0 required for a given process can be obtained from ω. As (50) indicates, ω depends not on crack and zone speed (c_0, c), but on

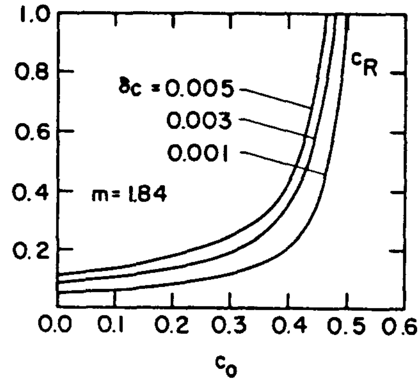

Fig. 3. ω vs non-dimensionalized crack speed.

crack speed and zone growth rate $(c_0, \delta c)$. Figure 3 shows that ω generally increases with both. That is, for a given ratio σ_0/Y, satisfaction of (66) requires that the faster the crack moves, the lower the zone growth rate must be. Consequently, (65) shows that, for a given crack speed, θ_0 varies inversely with zone growth rate. This effect confirms the Zehnder–Rosakis (1991) observation that concentrating the heat source mechanisms (the zone here) serves to increase the local temperature.

For further illustration, we consider a generic steel material with properties

$$\beta = -8.2(10^{-5})\frac{1}{0_K}, \quad v_l = 5900\frac{m}{s}, \quad m = 1.84, \quad \frac{Y}{\mu} = 0.001, \quad h = 0.00167\,\mu\text{m} \tag{67}$$

and cracks with rudimentary zones $\delta c \ll c$. In addition, precisely because the zone is small, we assume that the non-thermal σ_0/Y-relation is essentially correct, and consider

$$\delta\sigma = \sigma_0 - \omega Y = 0.0001\mu \tag{68}$$

in order to satisfy (66). That is, the applied stress σ_0 must exceed the non-thermal critical value by an amount $\delta\sigma$ which is $0(10^3)$ psi. Plots of θ_0 in Fig. 4 for (67), (68) and various values of $\delta c \ll c$ show an extremely rapid rise followed by a leveling-off at values after 1 ns that are of the order of magnitude noted by Zehnder and Rosakis (1991). Particular combinations of (σ_0, c_0) values for the δc-cases shown can be determined from (68) and Fig. 3. Equation (65) is, of course, valid only for a short time after fracture/zone initiation; the time range covered in Fig. 4 is chosen only to clearly illustrate the leveling-off of the θ_0-increase. For example, a value of $c_0 = 0.2$ in the material here would produce a crack extension of 1.18 mm by 1 μs after difraction. Finally, (65) and both Figs 3 and 4 indicate that the zero-speed $(c_0 = 0)$ limit case of pre-fracture inelastic zone growth shows little distinctive behavior.

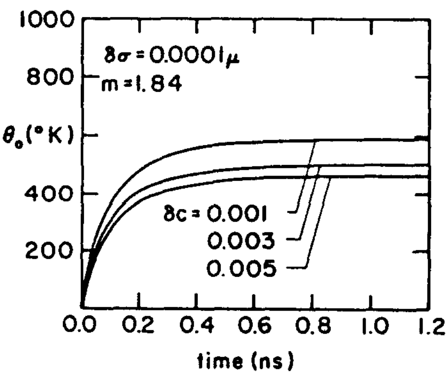

Fig. 4. Temperature rise vs time after diffraction.

Condition (66) is not itself a fracture criterion, although it could be incorporated in one. For insight into the fracture mechanics of the process considered here, we now study parameters that are often used in fracture criteria.

7. FRACTURE CRITERION PARAMETERS

In the criteria for Dugdale (1960) models, two parameters that are often used are the crack opening d_c at the crack edge (here, the tail of the inelastic zone) and the rate of energy production in the inelastic zone (Achenbach and Brock, 1971) per unit length of crack edge, $\dot{W}$. Returning now to the solution (39), we have in view of (34) and (40),

$$U_-^* = \frac{m^2 a_-}{\mu\lambda p^2 G_-(q-n_+)}\left[\frac{\Sigma}{n_- G_+(0)q} - \frac{a_+(k)}{G_+(k)(k-n_-)} \frac{Y}{p(k-q)}\right] + \frac{\beta\Psi(1-cq)^2 N}{p^3 R(\alpha^2-a^2)}\left(\frac{a}{\alpha}-1\right)\frac{1}{k-q} - \frac{\beta\Psi a_- P_+}{\lambda p^3 G_-(q-n_+)}. \quad (69)$$

Examination of (69) shows that the Σ-term is analytic for $\mathrm{Re}\,(q) < 0$, has a pole $q = 0$ and a branch cut $\mathrm{Im}\,(q) = 0$, $\mathrm{Re}\,(q) > i_+$. The Y-term is similar, except that its singularity, at $q = k$, lies on the branch cut. The first (a/α) Ψ-term has for large p branch cuts $\mathrm{Im}\,(q) = 0$, $m_- < \mathrm{Re}\,(q) < \tau_-$ and $\mathrm{Im}\,(q) = 0$, $\tau_+ < \mathrm{Re}\,(q) < m_+$ and poles at $q = (n_\pm, k, (1-\tau)/c)$. The second Ψ-term has the same poles for large p, but its branch cuts are $\mathrm{Im}\,(q) = 0$, $m_- < \mathrm{Re}\,(q) < i_-$ and $\mathrm{Im}\,(q) = 0$, $i_+ < \mathrm{Re}\,(q) < m_+$. The final term in (69) has a pole at $q = n_+$, and the branch cuts $\mathrm{Im}\,(q) = 0$, $m_- < \mathrm{Re}\,(q) < \tau_-$ and $\mathrm{Im}\,(q) = 0$, $\mathrm{Re}\,(q) > i_+$. Thus, the substitution of (69) into (47a) for $\xi < 0$ and the use of the Cauchy residue theory gives an expression for U_- in terms of integration around the right-hand branch cuts of the various integrands, and residues from those poles in the region $\mathrm{Re}\,(q) \geqslant 0$. Substitution of (51) into U_- and allowing $hp \sim \infty$ gives a transform suitable for the period right after fracture. Then, by using (29a,b), (43) and the same type of integration variable changes used to produce (41), we have

$$\mu U_- \sim \frac{1}{m^2 p}\left(\omega\frac{Y}{p} - \Sigma\right)e^{(\xi\sqrt{hp})} + \frac{\Sigma}{m^2 p} + \frac{m^2\omega Y}{p^2 n F_-(0)} P_{12}(\xi) \quad (70)$$

where

$$\pi P_{12}(\xi) = \left(\int_1^{c} N_0^2 + \int_m^{c} 4w^2|a_0 b_0|\right)\frac{(w+n)\sqrt{(1+cw)}}{w(1+c_0 w)D} F_+ \, e^{pw\xi/(1+cw)}\, \mathrm{d}w \quad (71)$$

and w-dependence is understood. Equation (70) can be inverted by using tabulated results (Carrier and Pearson, 1988). In particular, we have

$$\mu m^2 u_-(\xi, s) \sim \int_0^s [\sigma(s-t) - \omega Y]\,\mathrm{erf}\left(\frac{|\xi|}{2\sqrt{(ht)}}\right)\mathrm{d}t \quad (72)$$

for $-s < k\xi < 0$ ($c_0 s < x < cs$). The region of x considered corresponds to the inelastic zone itself. By symmetry, then,

$$d_c = 2u_-(-s/k, s). \quad (73)$$

This region of x also has the advantage that the w-integration of the inverse of the P_{12}-term can be performed by the Cauchy residue theory, *cf.* (70). In any case, (72) demonstrates that d_c does not depend on the thermoelastic parameter β, and vanishes both when $s = 0$ and, in accordance with (30d), when $\xi = 0$.

Turning now to the energy rate, it can be shown (Achenbach and Brock, 1971) that

$$\dot{W} = 2v_1 \int_{c_0 s}^{cs} Y\dot{u}_- \, dx. \tag{74}$$

Use of (14a), (72) and performing the x-integration in (74) gives

$$\frac{\mu m^2}{v_1 Y}\dot{W} \sim 2\delta c[\sigma(s) - \omega Y \operatorname{erf}(X)]s + c\int_0^s [\sigma(s-t) - \omega Y]\operatorname{erf}\left(X\sqrt{\left(\frac{s}{t}\right)}\right) dt$$
$$+ 2\omega Y(1 - e^{-X^2})\sqrt{\left(\frac{hs}{\pi}\right)} \tag{75}$$

where

$$X = \frac{\delta c}{2}\sqrt{\left(\frac{s}{h}\right)}. \tag{76}$$

Like d_c, $\dot{W}$ does not depend on β, and vanishes at $s = 0$.

For insight into the sensitivity of the relation between the inelastic zone/crack speeds and these fracture mechanics parameters, we once again consider the case (64). Then, (73) and (75) give

$$\mu m^2 d_c \sim 2(\sigma_0 - \omega Y)\int_0^s \operatorname{erf}\left(X\sqrt{\left(\frac{s}{t}\right)}\right) dt \tag{77a}$$

$$\frac{\mu m^2}{v_1 Y}\dot{W} \sim 2\delta c[\sigma_0 - \omega Y \operatorname{erf}(X)]s + c(\sigma_0 - \omega Y)\int_0^s \operatorname{erf}\left(X\sqrt{\left(\frac{s}{t}\right)}\right) dt + 2\omega Y(1 - e^{-X^2})\sqrt{\left(\frac{hs}{\pi}\right)}. \tag{77b}$$

Equations (77) both exhibit the factor seen in θ_0. Indeed, (66) must again hold for $d_c \geqslant 0$, and its satisfaction would certainly maximize $\dot{W}$. Another effect of the weak coupling considered here is that $(d_c, \dot{W})$ are not, as they would be in a non-thermal study, *cf.* Nuismer and Achenbach (1972), strictly linear in time (s). Another feature of (77b) is that $\dot{W}$ depends explicitly on zone speed (c), not just zone growth rate (δc).

In Figs 5 and 6, the dimensionless quantities (d_c/h, $\dot{W}/\mu v_1 h$) are plotted vs real time for (67), (68) and some allowable combinations of (c_0, δc). In view of the size of h, Fig. 5 shows that that the inelastic zone trailing edge (crack edge) separation is micromechanical in scale for some time after the supposed fracture. This is perhaps consistent with the Dugdale

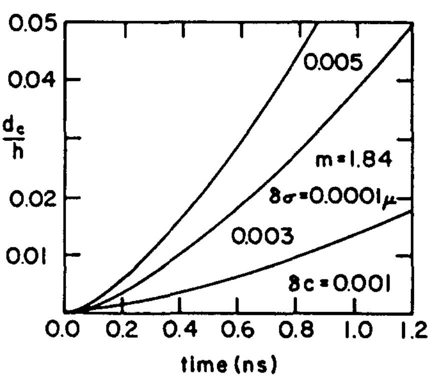

Fig. 5. Crack opening vs time after diffraction.

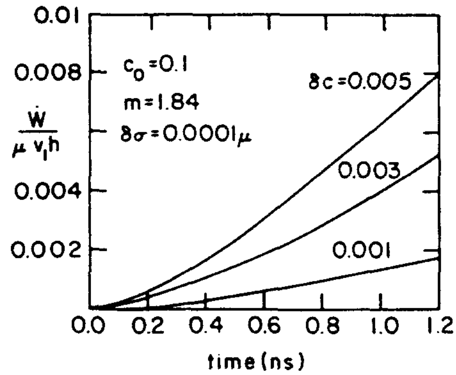

Fig. 6. Inelastic zone energy rate production vs time after diffraction.

model and small zone growth rates considered here. Even with (68) imposed, $\dot{W}$ shows noticeable variation with c_0, so several curve sets are presented in Fig. 6. It is noted that, as opposed to θ_0, both $(d_c, \dot{W})$ vary directly with zone growth rate δc.

In closing this section, it should be noted that dropping (68) and systematically varying (c, c_0) for the case $\sigma_0/Y \leqslant 0.1$ indicated that $(d_c, \dot{W})$ generally had the same sign. That is, crack opening appropriately requires energy production, at least right after fracture initiation.

8. DISCUSSION

This study considered thermal effects associated with rapid, nearly brittle crack propagation under stress wave diffraction. The cracked material was a weakly-coupled thermoelastic solid, and the crack edge exhibited a rudimentary inelastic zone of the Dugdale type, that also served as a heat flux site. A transient two-dimensional analysis valid for the period immediately after fracture/zone initiation gave expressions for the zone heat flux, zone edge temperature change, crack opening and the rate of energy production in the zone.

Beyond the inelastic crack-blunting effect, no particular fracture criterion was imposed, but insight was gained by studying the expressions themselves: for the diffraction of an incident step-stress wave, it was found that the incident wave stress level, the yield stress, crack speed and zone growth rate must satisfy a certain inequality. Because the integral transform of the temperature field in the solid showed the typical exponential decay with distance from the source (the zone), the temperature at the zone edge itself was examined. It was found to increase rapidly, then level off with time, and to vary inversely with zone growth rate. Qualitative agreement with more detailed steady-state results was also noted. The crack opening and energy rate increased continuously with time, and varied directly with zone growth rate.

Because we were interested in nearly brittle fracture, zone growth rates were kept small in computations, in keeping with the vanishing thickness of the zone. Moreover, incident wave stresses that barely satisfied the aforementioned inequality were employed in computations—a strict equality being the non-thermal limit case. The aforementioned temperature behavior was, therefore, gratifying, since Parvin (1979) has shown that zone thickness should generally not be ignored in temperature calculations, while Zehnder and Rosakis (1991) have surmised that a Dugdale zone artificially concentrates heat source mechanisms.

The limitations of the model employed here went beyond zone geometry: in particular, the crack and inelastic zone were assumed to grow simultaneously at constant speeds, immediately upon stress wave diffraction or, in the limit case of no fracture, zone initiation was instantaneous. The yield stress temperature variation, as well as that of other material constants, was neglected. The inelastic zone heat flux function, while not *a priori* given, was assumed, as a first step, to vary only with distance from the crack edge.

The other basic limitation, that of a weakly-coupled thermoelastic solid, was perhaps less restrictive: the dimensionless thermoelastic coupling constant, known to be $\ll 1$ for many materials, was simply treated as a negligible perturbation in sums with dimensionless numbers of 0(1). The resulting field equations were generally valid for long and short times after fracture/zone initiation. Indeed, the equations were analogous to fully-coupled static thermoelasticity.

The crack-blunting requirement gave a relation between the flux function transform and transform functions related to the yield stress and incident wave stress. An alternative possibility, for brittle fracture, would be to require that the intensity factor achieve a fracture criterion-specified behavior. This would again allow a flux function relation. In either case, this function clearly serves as a useful characterization of both thermal and mechanical response in the crack edge region, especially when an idealized model is used to represent what can be complicated local thermo-mechanical behavior.

In performing the study, integral double transforms and the Wiener–Hopf technique were employed. In general, exact expressions for the various field variable transforms were possible, despite the presence of a characteristic length that caused the regions of analyticity in the spatial transform plane to vary with the time transform variable. Only upon inversion were approximations used and these were valid for the short time ranges considered.

In summary, then, this study did rely on certain idealizations that are not used in other work cited here. Nor did it provide a complete, thermodynamically-based, fracture mechanics model. As the work of Zehnder and Rosakis (1991) shows, numerical schemes capable of handling more realistic inelastic zone geometries are probably ultimately necessary. However, this study did, as a first step, provide transient results for dynamic loading and rudimentary inelastic zones, and did allow thermal effects in the elastic material surrounding the zone/crack. It serves, therefore, as a basis for further work, and as a limit-case check on more ambitious transient studies.

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REFERENCES

Achenbach, J. D. (1973). *Wave Propagation in Elastic Solids.* North-Holland, Amsterdam/Elsevier, New York.

Achenbach, J. D. and Brock, L. M. (1971). Rapid extension of a crack. *J. Elasticity* **1**, 51–63.

Boley, B. and Tolins, I. S. (1962). Transient coupled thermoelastic boundary-value problems in the half-space. *ASME J. Appl. Mech.* **29**, 637–646.

Boley, B. and Weiner, J. H. (1960). *Theory of Thermal Stresses.* Wiley, New York.

Boyer, H. E. and Gall, T. L. (1985). *Metals Handbook.* American Society for Metals, Ohio.

Carrier, G. F. and Pearson, C. E. (1988). *Partial Differential Equations*, 2nd edn. Academic Press, New York.

Chadwick, P. (1960). Thermoelasticity, the dynamical theory. In *Progress in Solid Mechanics* (Edited by I. N. Sneddon and R. Hill), Vol. 1, pp. 263–328. North-Holland, Amsterdam.

Dugdale, D. S. (1960). Yielding of steel sheets containing slits. *J. Mech. Phys. Solids* **8**, 100–104.

Freund, L. B. (1971). Crack propagation in an elastic solid subjected to general loading—I: constant rate of extension. *J. Mech. Phys. Solids* **20**, 129–140.

Gradshteyn, I. S. and Ryzhik, I. M. (1980). *Table of Integrals, Series and Products.* Academic Press, New York.

de Hoop, A. T. (1960). A modification of Cagniard's method for solving seismic pulse problems. *Appl. Sci. Res.* **B8**, 349–360.

Noble, B. (1958). *Methods based on the Wiener–Hopf Technique.* Pergamon Press, New York.

Nuismer, R. J. and Achenbach, J. D. (1972). Dynamically induced fracture. *J. Mech. Phys. Solids* **20**, 203–222.

Parvin, M. (1979). Theoretical prediction of temperature rise at the tip of a running crack. *Int. J. Fract.* **15**, 397–404.

Peirce, B. O. and Foster, R. M. (1957). *A Short Table of Integrals.* Blaisdell, Massachusetts.

Rice, J. R. and Levy, N. (1969). Local heating by plastic deformation at a crack tip. In *Physics of Strength and Plasticity* (Edited by A. S. Argon). MIT Press, Cambridge, Massachusetts.

Sneddon, I. N. (1972). *The Use of Integral Transforms.* McGraw-Hill, New York.

Sneddon, I. and Lowengrub, M. (1969). *Crack Problems in the Classical Theory of Elasticity.* Wiley, New York.

Taylor, G. I. and Quinney, M. A. (1934). The latent energy remaining in a metal after cold working. *Proc. Roy. Soc.* **A143**, 307–322.

Weichert, R. and Schoenert, K. (1974). On the temperatrure rise at the tip of a fast running crack. *J. Mech. Phys. Solids* **22**, 127–133.

Williams, J. G. (1972). Visco-elastic and thermal effects on crack growth in PMMA. *Int. J. Fract. Mech.* **8**, 393–401.

Zehnder, A. T. and Rosakis, A. J. (1991). On the temperature distribution at the vicinity of dynamically propagating cracks in 4340 steel. *J. Mech. Phys. Solids* **39**, 385–415.